14·4521 grm. in mass, of a constant cross-sectional area, and of a length of 106·300 cm., while an ohm has a resistance of 10^9 cm./sec.

The agreement is most satisfactory.

The conclusion is that a resistance of 1 International ohm is equal to $1·00052 \pm 0·00004$ ohm (10^9 cm./sec.), the probable error of $\pm 0·00004$ being approximately the sum of those involved in the resistance of the ohm and the International ohm. It follows that a column of mercury, at 0° C., $106·245 \pm 0·004$ cm. long, of constant cross-sectional area (the same as that of the International ohm), has a resistance of 1 ohm. The mass of this column will be $14·4446 \pm 0·0006$ grm.

I desire to express my thanks to the Drapers' Company of London and to Sir Andrew Noble for their generous help, to Lord Rayleigh for his keen interest in the work, and to Dr. Glazebrook for his very valuable help and advice throughout the investigation.

Elastic Hysteresis in Steel.

By F. E. Rowett (Whitworth Scholar), Research Student of the University of Cambridge.

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The rate of dissipation of energy by internal molecular friction, when steel undergoes variations of stress within what is ordinarily regarded as the elastic limit, was the subject of a recent paper by Hopkinson and Trevor-Williams.* A bar of steel having an elastic range of about 25 tons per square inch was subjected to direct (axial) push and pull, the limits of tension and compression being equal, and the frequency of the cycles about 120 per second. The dissipation of energy by elastic hysteresis was determined by the fall of temperature between the middle point of the bar and the ends. It could be measured accurately in this way when the range of stress was near the elastic limit (25 tons per square inch) and could be detected when the range was 10 tons per square inch.

One object of the research just referred to was to ascertain whether the dissipation of energy per cycle of stress increased with the speed of reversal, in other words whether the internal friction to which this dissipation is due is of the nature of solid friction, or is more analogous to the viscosity of fluids. For this purpose the bar was alternately compressed and extended, statically, in a testing machine, between the same limits of stress as in the high-speed tests, and the stress-strain curve was obtained by means of a specially designed extensometer.

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* 'Roy. Soc. Proc.,' A, vol. 87, p. 502.

The curve is of course a very narrow loop, the area of which corresponds to the dissipation of energy in the slow-speed cycle. It was not found possible to do more than determine the width of this loop at the origin, corresponding to the difference in length of the unstressed piece according as the last application of stress had been compression or tension. The area of the loop is equal to the product of this difference into the range of strain, multiplied by a constant which depends on the shape of the loop. In the paper under consideration it was assumed that the loop was of lenticular form, being bounded by two arcs of circles of large radius, and on that assumption the hysteresis in the static test was at all stress ranges about 1·25 times that found in the high-speed cycle between the same limits of stress. As pointed out in the paper, the uncertainty as to the shape of the loop precluded any very definite inference as to the effect of speed of reversal on hysteresis except that the difference, if any, was probably in the direction of reduced hysteresis at high speeds, and that it was not more than 30 per cent. as between 120 cycles per second and quite slow reversals.

The present research is a further study of this problem. In order to secure higher accuracy in the measurement of the static hysteresis, thin-walled tubes have been used in torsion. By the use of spirit levels in the manner described in greater detail below, it has been found possible to detect shearing strains of the order of 10^{-6}, corresponding to a stress of 12 lb. per square inch, and to determine within about 5 per cent. the energy dissipated by elastic hysteresis in a cycle of stress well within the elastic limit. The corresponding high-speed cycles were observed by the method of torsional oscillations, first employed by Lord Kelvin. One end of the tube was fixed, and a fly-wheel was mounted on the other end, of such inertia as to give a frequency of torsional vibration of about 70 per second, and the amplitude of vibration was recorded photographically. From the rate of decay, after correcting for air-friction, the hysteresis loss is calculated.

Briefly, the result of these experiments is to show that the hysteresis is, probably within 5 per cent., the same at high speeds as at low speeds. Furthermore, the shape of the hysteresis loop, which was left uncertain by Hopkinson and Trevor-Williams' work, has been determined. If the constant giving the area of the loop from its principal dimensions, which has been determined by the present author for torsional cycles, be substituted for the conjectural constant used by them, the result is to bring the hysteresis of their static cycles into almost exact agreement with that which they found at high speeds.

It also appears that in a hard drawn tube as supplied by the makers the hysteresis is for all stress-ranges only about one-eighth of the hysteresis in the same tube after it has been softened by annealing. In the annealed tube the loss per cubic centimetre per cycle, with a stress-range in shear of 1160×10^6 dynes per square centimetre (7·5 tons per square inch), is about 11,000 ergs, and it varies approximately as the cube of the stress-range.

Hopkinson and Trevor-Williams found at the corresponding range of stress a loss of about 4000 ergs in a steel bar which had not been annealed, and at higher stresses the loss in their experiments varied as the fourth power of the stress-range.

High-speed Cycles.

The material was in the form of tubes having a mean external diameter of 0·625 inch, and a mean thickness, determined by weighing, of 0·0475 inch.

The steel had the following composition:—Carbon, 0·17 per cent.; manganese, 0·24 per cent.; sulphur, 0·02 per cent.; phosphorus trace, the remainder being iron.

The tubes, as they came from the makers, were hard, as the result of the drawing. The elastic limit in torsion in this state was 13·8 tons per square inch shear stress in either direction, giving an elastic range of 27·6 tons in shear or 55 tons in tension and compression. By annealing at 800° C. for fifteen minutes and cooling in the furnace the effect of the drawing was removed. The tubes were then quite soft and ductile and had a well-marked yield point in torsion of 5·58 tons per square inch shearing stress, the elasticity as determined by ordinary tests being nearly perfect up to that stress. The elastic range in this condition is, therefore, roughly 11·2 tons per square inch in shear or 22·4 tons per square inch in tension and compression.

The specimen tube AB was firmly fixed at B, fig. 1 (thorough and smooth soldering was found to be efficient), into a steel block, which in turn was securely bolted down to a cast-iron frame of very massive proportions. The upper end of the tube was fixed into a small flywheel, the whole being arranged with the tube vertical, as shown in fig. 1. Four very small stops were fitted into the flywheel, two being utilised for applying a pure torque by means of stirrups E and F. Cords passed from the stirrups over pulleys fitted with ball-bearings, and were attached to scale pans G and H.

The two columns M and N were fitted with adjustable centres, about which the ring PQ could be rotated. This ring carried two stiff, fine-threaded screws which engaged with the remaining two stops on the flywheel. The latter were fitted so that the upper edge of one and the lower edge of the other coincided with each other and with the centre of the flywheel. The heights of the adjustable centres were set so that upon rotating the ring the screws left the stops, the one at the upper edge and the other at the lower edge, at the same instant.

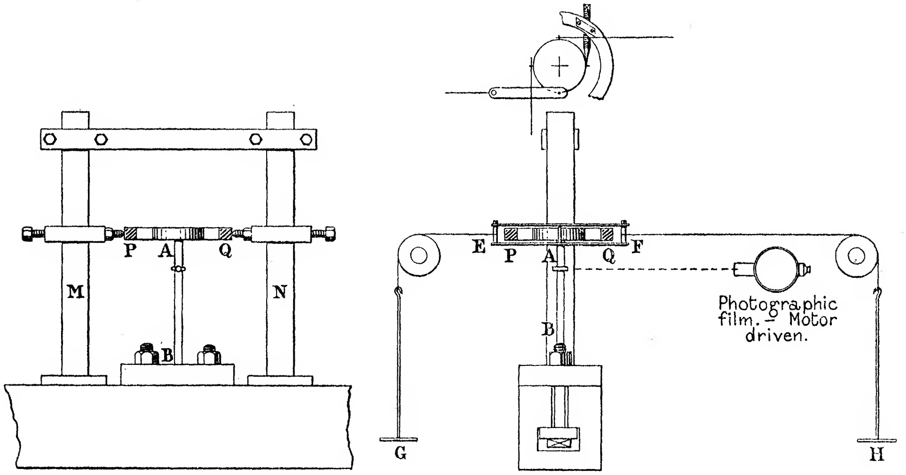


Fig. 1.

A mirror was attached to the tube at a definite distance from the fixed end, and records were taken of its motion upon motor-driven photographic films. In making an experiment, a definite torque was applied to the tube by means of weights and stirrups, and the screws adjusted so as just to take the stress on turning the ring into position.

This could be conveniently done by observing the spot of light, any movement of the spot on turning the ring indicating a faulty adjustment. The weights and stirrups were now removed without disturbing the distribution of stress in the tube.

A record was finally obtained by rotating the ring clear of the stops and thus releasing the flywheel, which was thereby set in torsional vibration.

A calibration was taken upon the same film by applying the same torque and obtaining the deflection in the form of a continuous line on the film, the ring meanwhile remaining free, so that the calibration was independent of the setting or the adjustments.

It was found that 1 cm. deflection corresponded to a stress of 229×10^6 dynes per square centimetre, *i.e.* about 1·48 tons per square inch.

The effect of air friction was investigated by fixing a small steel post above the flywheel to which was attached a disc of thin sheet tin. Films were obtained with the tin disc attached and also without the tin disc for the same specimen, and the increased damping obtained by difference. The air-friction was found to be small by comparison when the steel tube used had been annealed. The tubes, however, as supplied were in a hardened state due to the process of manufacture, and in this condition the dissipation of energy in the specimen was found to be considerably less than in the annealed state. Advantage was taken of this fact to determine the energy lost by air friction.

The friction loss is due almost entirely to the vibration of the flywheel, the motion of the tube being very small indeed. The flywheel was a plain smoothly turned disc, and the friction loss per cycle was assumed to be the same as that of a disc of the same diameter of tin plate with a rim attached, the width of the rim being equal to half the thickness of the flywheel. The disc was attached at a distance sufficiently away from the flywheel to avoid any interaction, and the two had exactly the same motion.

The weight of the disc was so small in comparison to the flywheel that the frequency of vibrations was altered by only a very small amount (about 0·25 per cent.). The damping curves obtained are shown in fig. 2. The amount of energy stored in the flywheel and tube per cubic centimetre of steel is equal to the (stress-range) $^2/8$ C ergs, where C is the modulus of rigidity, the stress-range and C being both expressed in dynes per square centimetre.

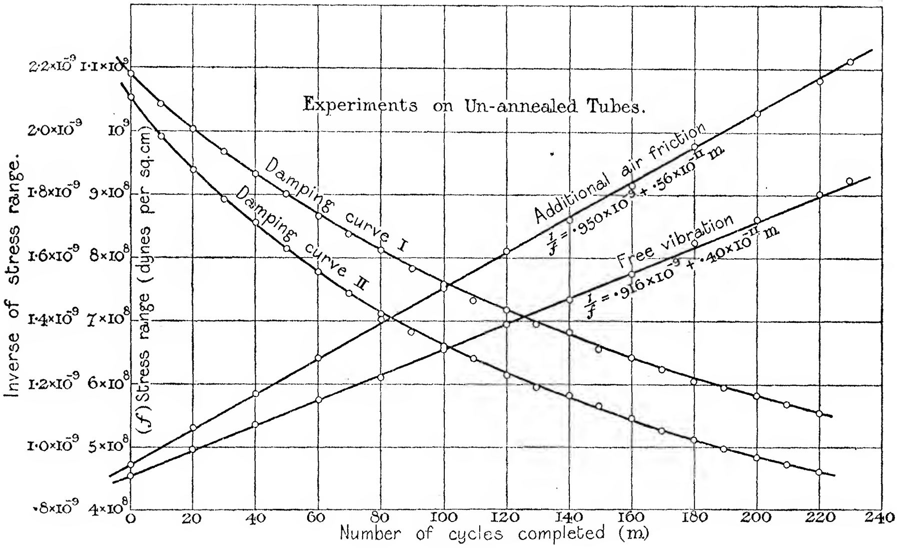


Fig. 2.

Let x = amplitude, which is proportional to the range of stress, and suppose the energy lost per cycle $\propto x^m$. Then the loss of energy per second is given by

$$nx^m \times A = Bx\dot{x},$$

where n = number of cycles per second and A and B are constants; the integral of which is

$$t = \frac{B}{nA} \cdot \frac{1}{(2-m)} \{x^{2-m} - x_0^{2-m}\},$$

where x_0 is the amplitude when $t = 0$.

If $m = 3$, corresponding to the law friction $\propto$ velocity^2, the law of variation of amplitude is given by

$$t = \lambda\left[\frac{1}{x} - \frac{1}{x_0}\right], \text{ where } \lambda = \text{constant},$$

that is the inverse of the stress-range plotted against the number of cycles performed should give a straight line. The latter has been done for the curves above and the results are shown plotted in fig. 2. Straight lines were obtained both for free vibrations and also when the additional air disc was attached.

The equations of the lines were found to be

I. Free vibration............... $1/f = 0{\cdot}916 \times 10^{-9} + 0{\cdot}40 \times 10^{-11} m$,
II. Additional air friction ... $1/f = 0{\cdot}950 \times 10^{-9} + 0{\cdot}56 \times 10^{-11} m$,

where f = stress range in dynes per square centimetre, and
m = the number of cycles performed.

The energy present per cubic centimetre of metal $= \frac{1}{8} f^2/C$ ergs, where C = modulus of rigidity.

From Equation I $\quad -\frac{1}{f} \cdot \frac{df}{dm} = 0{\cdot}40 \times 10^{-11}$,

" " II $\quad -\frac{1}{f} \cdot \frac{df}{dm} = 0{\cdot}56 \times 10^{-11}$.

The energy lost per cycle per cubic centimetre $= -\frac{f}{4C} \cdot \frac{df}{dm}$ ergs.

$C = 82{\cdot}8 \times 10^{10}$ dynes per square centimetre (12×10^6 lb. per square inch).

Therefore from I the energy lost per cubic centimetre per cycle is

$$1{\cdot}208 \times 10^{-24} \times f^3 \text{ ergs},$$

and from II $\quad 1{\cdot}691 \times 10^{-24} \times f^3$ "

The energy in ergs per cubic centimetre per cycle lost by air friction is therefore

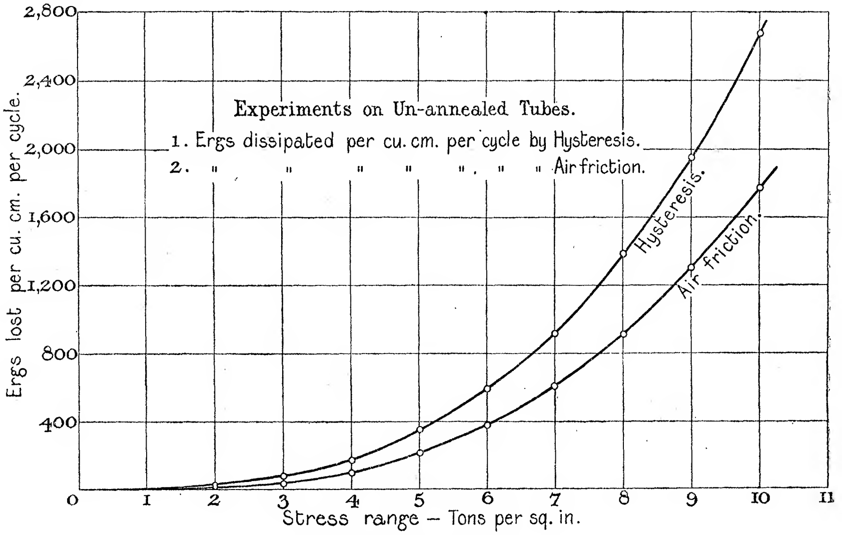
$$0{\cdot}483 \times 10^{-24} \times f^3 \text{ ergs},$$

and by hysteresis $\quad 0{\cdot}725 \times 10^{-24} \times f^3$ "

f being in dynes per square centimetre. If f be expressed in tons per square inch,

$$\text{Hysteresis loss} = 2{\cdot}677 \times f^3 \text{ ergs per cubic centimetre per cycle.}$$
$$\text{Air frictional loss} = 1{\cdot}783 \times f^3 \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

Fig. 3 shows the results plotted. The frequency of the vibrations in the above experiments was 67 cycles per second.



67 cycles per second.
Fig. 3.

Experiments with Annealed Tubes.

Some of the tubes employed were annealed by gradually heating them throughout their length to a temperature of 800° C in an electric furnace, maintaining this temperature for about 15 minutes, and then allowing them to cool in the furnace. The cooling took about 4 hours.

For the same stress-range the damping was found to have considerably increased and Table I gives the dissipation in ergs per cycle at the various stress-ranges, while fig. 4 shows results plotted. The length of the tube employed was the same as in the previous experiments and the frequency was found to be the same. Column 7 gives the calculated dissipation due to air friction and the hysteresis loss is obtained by difference.

It will be seen from fig. 4 that at a stress-range of 1160×10^6 dynes per square cm. (7·5 tons per square inch) the hysteresis loss per cubic centimetre per cycle is 11,000 ergs, whereas the air friction loss is 740 ergs or about 6·7 per cent.

Table I.—Experiments on Annealed Tubes. Annealed at 800° C. and Cooled in Furnace. Vibrations 67 per sec.

No. of cycles.	Amplitude.	Stress-range.	Energy.	Mean stress.	Dissipation.	Correction, air friction.	Hysteresis.
	cm.	tons per sq. in.	ergs.		per cycle.		ergs per cycle.
0	5·17	7·82	$2·201 \times 10^5$	7·60	11,950	770	11,180
2	4·88	7·38	$1·962 \times 10^5$	7·19	10,050	660	9,390
4	4·62	6·99	$1·761 \times 10^5$	6·83	8,050	560	7,490
6	4·41	6·67	$1·600 \times 10^5$	6·51	7,200	480	6,720
8	4·21	6·36	$1·456 \times 10^5$	6·22	6,450	417	6,033
10	4·02	6·08	$1·327 \times 10^5$	5·93	5,950	360	5,590
12	3·83	5·79	$1·208 \times 10^5$	5·66	5,300	312	4,988
14	3·66	5·54	$1·102 \times 10^5$	5·42	4,650	280	4,370
16	3·50	5·30	$1·009 \times 10^5$	5·17	4,550	236	4,314
18	3·34	5·05	$0·918 \times 10^5$	4·94	3,950	200	3,750
20	3·19	4·83	$0·839 \times 10^5$	4·39	2,800	132	2,668
30	2·62	3·94	$0·559 \times 10^5$	3·57	1,911	76	1,835
40	2·11	3·20	$0·368 \times 10^5$	2·91	1,221	44	1,177
50	1·73	2·62	$0·246 \times 10^5$				

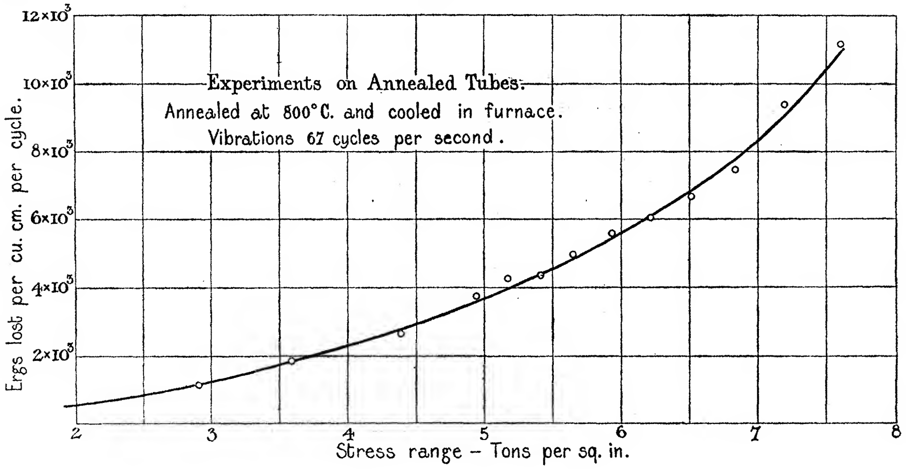


Fig. 4.

Order of Accuracy in Dynamical Experiment.

The amplitudes of the vibrations upon the photographic films could be measured to 1/5 mm., so that, at the maximum range of about 5 cm., the possible error did not exceed 1/250 in the amplitude or 1/125 in the energy. This amounts to 1600 ergs per cubic centimetre.

The dissipation is calculated by obtaining the energy present every two complete cycles, taking the difference and dividing by two.

The difference at the higher ranges amounts to about 20,000 ergs, so that the possible error in the measurement of energy dissipated in any one cycle does not exceed 8 per cent.

In the fair curve drawn through the plotted points for this quantity in terms of the number of cycles completed, the accidental errors of observation disappear in large measure and the error at the higher stresses probably does not exceed 4 per cent.

Static Experiments.

Static tests were performed upon the tubes used in the dynamical experiments, with the object of finding the actual shapes and areas of the hysteresis loops for various stress-ranges when these were performed slowly by hand.

The method employed was to submit the tube to cyclic stress-ranges performed in the direction of the arrows, fig. 5. After a number of cycles the process was stopped at a point E and the strain measured. The cycle was then continued until the point F was reached having exactly the same stress but being in the opposite portion of the loop.

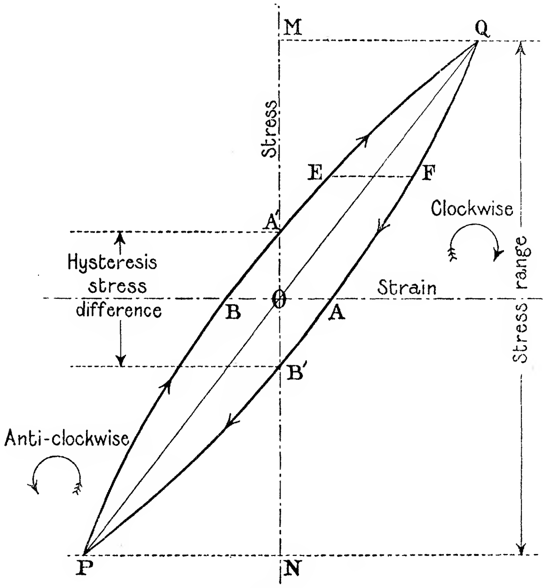


Fig. 5.

The strain was again measured and the difference between the then value and the previous value for the same stress gave the length EF.

By varying the position of the points E and F and meanwhile keeping the stress-range constant the exact shape of the loop for this range was obtained.

The difficulty in such experiments of eliminating completely the effects of friction and hysteresis in the apparatus which is used for measuring stress was overcome in this case by fixing the tube under experiment at one end to a much longer and stronger hard drawn tube, through which the torque was applied, and by the twist of which it was measured. The dimensions of the tube were such that the stress in it was about 1/7th of that in the small tube.

Since the hysteresis stress difference (measured by A′B′, fig. 5) varies approximately as the square of the stress-range, the value of the stress difference in the large tube was about 2 per cent. of its value in the small tube. The hysteresis in the large tube was removed by applying a small torque until the actual twist in the large tube was zero. This small torque, however, removed some of the twist due to hysteresis in the small tube, and the hysteresis observed in the small tube was thus smaller than the actual value by 14 per cent. when both tubes were in the hard drawn condition. When the small tube was annealed and the other hard drawn the error was about 1·8 per cent.

Static Apparatus.

The stiff tube was rigidly fixed at one end and the torque was transmitted to it through the experimental tube. The lengths of the tubes were such that the angular twist in each was approximately the same. Fig. 6 gives a general view and details of the apparatus. The compound tube GH was rigidly fixed at G, while the end H was keyed to lever L working between adjustable stops E and F. Sensitive micrometer levels, fig. 6 (*a*), were fixed to the tubes at the positions A, B, C, and D, and at M_1 and M_2 small plane mirrors, fig. 6 (*b*), were arranged so that reflections from scales S_1 and S_2 were observed in telescopes T_1 and T_2 respectively.

The weights W exactly balance the weight of lever L. The mirrors M_1 and M_2, in conjunction with the telescopes and scales, supply the means of measuring the extreme ranges of strain, and thus of stress.

In making an experiment the desired torque is applied to lever L, fig. 6, and by means of the micrometer screws all four levels are adjusted so that the bubbles read zero. The desired cycle of stress is then applied by movement of lever, which is returned approximately to its original position, and there adjusted by means of screw P until the two levels A and B again read exactly zero.

If the hysteresis in the large tube is neglected the torque is now precisely the same as before. The levels C and D will now no longer read zero, and the difference between them represents the hysteresis corresponding to the cycle of stress which has been performed (*i.e.* length EF in fig. 5).

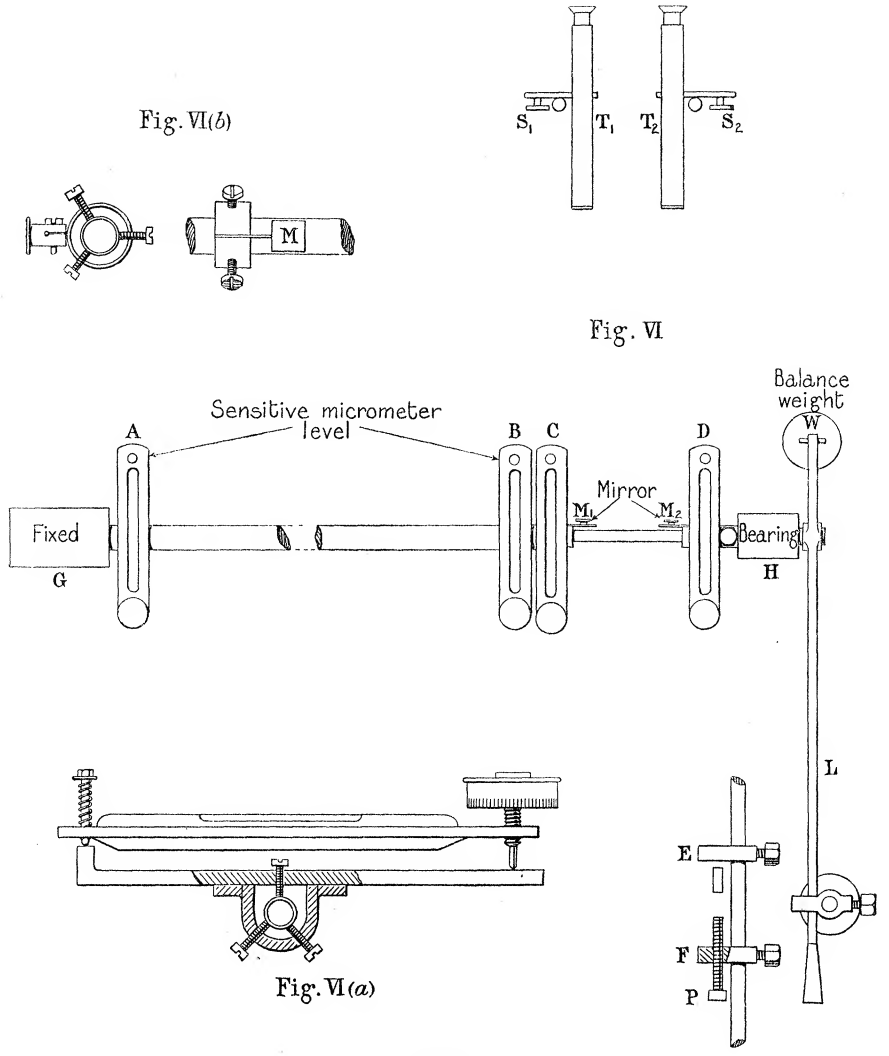


Fig. 6.

Shape of the Loop.

To obtain a comparison between the hysteresis at high frequency and that obtained by static experiments, it is necessary to know the area of the loop, that is to obtain the relation between the maximum and mean widths.

This has been done in the following manner :—A definite stress-range was first decided on, and the stops E and F adjusted as required, the range being calculated as before from the mirror readings at the extreme positions of lever L.

After a few hundred cycles had been performed at this range, in order to get the material into a cyclic state, the process was stopped at a definite point in the cycle and the stress maintained by means of screw P. All the levels were now adjusted to read zero and the readings of the telescopes noted. From the latter and a knowledge of the readings for zero stress, the position in the cycle was ascertained. The cycle was now continued to one extreme position and the lever brought back and screw P adjusted until levels A and B were again zero. This occurs (neglecting hysteresis in the large tube) when the stress in the large tube, and therefore in the small tube, is exactly the same as before. The levels C and D were now found to have readings other than zero, showing that the strain in the tube is slightly different from what it was before.

Table II gives the results for a cycle covering a range of 7·86 tons per square inch (±3·93 tons). The last column but one gives the difference of twist observed at various points in the cycle, according as the stress is increasing or decreasing. The last column gives the corresponding stress difference, and is obtained by multiplying the strain in the preceding column by the modulus C. These results are shown graphically, the figure representing the hysteresis loop being magnified 40 times in the direction of the stress axis. It should be noted that in setting out the figure it is assumed that the observed stress difference, such as RS, fig. 7, is bisected by the line PQ which corresponds to perfect elasticity. This assumption is, of course, not necessarily true, and to this extent the real shape of the loop is uncertain, but it does not affect the area of the loop, which it is the main object of these experiments to determine. The mean width of the loop is 0·506 of the maximum width, and the area in ergs of the cycle is to be obtained by multiplying the maximum width on the stress axis by the range of strain and by the constant 0·506. Hopkinson and Trevor-Williams assumed this constant to be 2/3, and the dissipation for high-speed cycles then appeared to be 0·8 times that observed in the statical experiments. The use of the factor 0·506 in reducing the results of the latter brings the two into close agreement. Another experiment at a range of 6 tons per square inch gave the same ratio of maximum to mean width within 1 per cent. At lower stress-ranges the accuracy is not sufficient to determine the shape of the loop but it may be assumed that it is substantially independent of the stress-range, and that the area may be calculated from the width of the loop at zero stress by the use of this same constant. This width was determined for a number of stress-ranges. The procedure was exactly the same as described above, the only points determined in each case, however, being those corresponding to zero or approximately zero stress.

The results are plotted in fig. 8. The corresponding loop areas calculated from the maximum widths by the use of the constant 0·506 gave when plotted the curve shown in fig. 9. On the same figure are shown the values of the energy dissipated per cycle at 67 cycles per second, as determined with the dynamic machine. It will be seen that the two agree very closely.

Table II.—Experiments on Annealed Tubes. Shape of Hysteresis Loop. Stress-range 7·86 tons per sq. in.

Mirror readings.		Equivalent stress.	Level readings.		Equivalent micrometer readings.		Difference.	Strain lag twist in radians.	Width of loop.
			C.	D.					
		Tons per sq. in.							Tons per sq. in.
11·32	9·80	3·56	0	0·4	0	0·31	0·31	0·033 × 10^{-3}	0·0055
12·32	11·13	3·05	0·05	1·6	0·04	1·25	1·21	0·129 × 10^{-3}	0·0216
13·32	12·47	2·595	0·1	2·75	0·07	2·14	2·07	0·221 × 10^{-3}	0·0370
14·32	13·80	2·20	0·1	3·80	0·07	2·97	2·90	0·309 × 10^{-3}	0·0517
15·32	15·11	1·772	0·2	5·0	0·14	3·90	3·76	0·401 × 10^{-3}	0·0672
16·32	16·46	1·303	0·3	6·4	0·22	5·00	4·78	0·510 × 10^{-3}	0·0853
17·32	17·77	0·878	0·3	7·3	0·22	5·61	5·47	0·584 × 10^{-3}	0·0977
18·32	19·11	0·401	0·4	8·1	0·29	6·32	6·03	0·643 × 10^{-3}	0·1075
19·33	20·42	Nil	0·4	8·8	0·29	6·86	6·57	0·701 × 10^{-3}	0·1172
20·34	21·72	0·398	0·4	8·1	0·29	6·32	6·03	0·643 × 10^{-3}	0·1075
21·34	23·06	0·874	0·3	7·4	0·22	5·77	5·55	0·594 × 10^{-3}	0·0995
22·34	24·37	1·293	0·3	6·4	0·22	5·00	4·78	0·510 × 10^{-3}	0·0853
23·34	25·72	1·77	0·2	5·1	0·14	3·98	3·84	0·409 × 10^{-3}	0·0683
24·34	27·03	2·19	0·1	3·8	0·07	2·97	2·90	0·309 × 10^{-3}	0·0517
25·34	28·36	2·606	0·1	2·8	0·07	2·18	2·11	0·225 × 10^{-3}	0·0376
26·34	29·70	3·10	0	1·5	0	1·17	1·17	0·125 × 10^{-3}	0·0209
27·34	31·03	3·58	0	0·4	0	0·31	0·31	0·033 × 10^{-3}	0·0055

Table III.—Experiments on Annealed Tubes. Annealed at 800° C. and Cooled in Furnace.

Setting mirror readings.			Equivalent stress-range.	No. of cycles.	Level readings.		Equivalent micrometer readings.		Difference.	Angle of twist, radians.	Hysteresis stress difference.	Area of cycle.
					C.	D.						
			Tons per sq. in.								Tons per sq. in.	ergs.
Top ...	29·22	31·98	8·53	100	0·4	11·0	0·29	8·57	8·28	0·844 × 10^{-3}	0·148	18,350
Bottom	10·32	6·79										
Top ...	28·40	30·88	7·67	100	0·4	8·2	0·29	6·39	6·10	0·651 × 10^{-3}	0·109	12,180
Bottom	11·32	8·11										
Top ...	27·80	30·10	7·22	100	0·3	7·2	0·22	5·61	5·39	0·575 × 10^{-3}	0·0964	10,120
Bottom	11·80	8·78										
Top ...	27·20	29·32	6·73	100	0·3	6·15	0·22	4·80	4·58	0·489 × 10^{-3}	0·0819	8,020
Bottom	12·33	9·50										
Top ...	26·79	28·75	6·51	100	0·2	5·5	0·14	4·29	4·15	0·442 × 10^{-3}	0·0741	7,020
Bottom	12·79	9·97										
Top ...	26·19	27·95	5·78	100	0·15	4·6	0·11	3·59	3·48	0·371 × 10^{-3}	0·0621	5,220
Bottom	13·37	10·89										
Top ...	24·81	26·13	4·61	100	0·1	3·2	0·07	2·50	2·43	0·259 × 10^{-3}	0·0434	2,900
Bottom	14·68	12·63										
Top ...	24·15	25·27	3·97	100	0·1	2·6	0·07	2·03	1·96	0·209 × 10^{-3}	0·0350	2,020
Bottom	15·37	13·57										

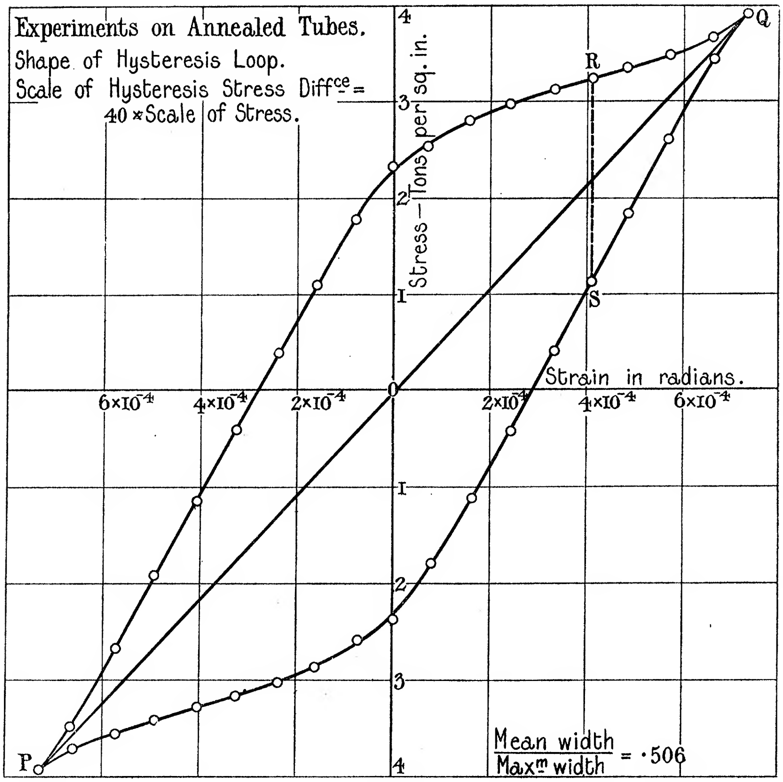


FIG. 7.

Unannealed Tubes.

An unannealed tube was introduced into the static apparatus and readings taken for the width at the mid-point of hysteresis loop.

It was found as might be expected from the high-speed experiments that the hysteresis was very small except at high ranges of stress. The following value is given as illustration :—

Stress-range, 8·52 tons per square inch ($13 \cdot 17 \times 10^8$ dynes per square centimetre).
Width of loop at centre, 0·015 ton per square inch.
Area of loop, 1820 ergs.

At the same stress-range the dissipation found in the high-speed machine was 1650 ergs (see fig. 3). The difference between this result and that obtained statically is within the errors of experiment.

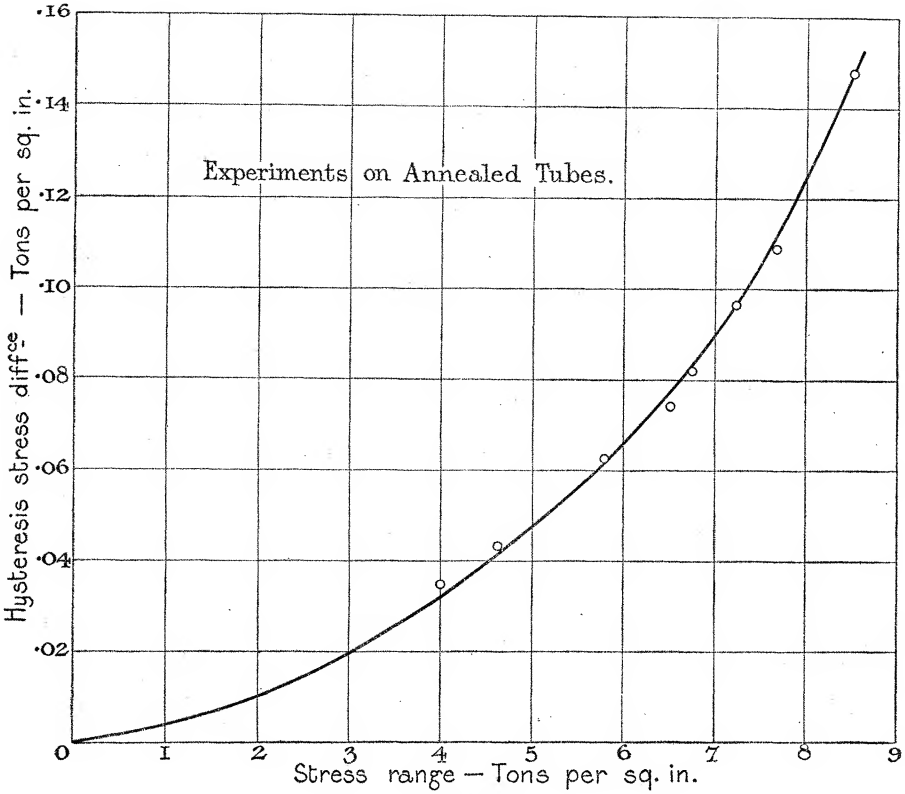


Fig. 8.

Order of Accuracy in the Static Experiments.

The levels used were placed 10 inches apart upon the experimental tube, and when in position the levels were calibrated against rotation of micrometer heads. The divisions on the levels were about $\frac{1}{8}$ inch long and readings of these could be easily estimated to $\frac{1}{5}$th of a division.

One division of the levels required, taking the mean value of levels C and D, 0·75 of a thousandth on the micrometer head.

Thus $$\tfrac{1}{5}\text{th division} = \frac{0{\cdot}15}{1000}\text{ in.}$$

The micrometer head worked upon a radius of $9\frac{3}{8}$ inches, so that the angle of twist could be accurately estimated to within

$$\frac{0{\cdot}15}{1000} \times \frac{8}{75} = \frac{8}{500{,}000}\text{ radians.}$$

The angle of shear $\phi = r\theta/l$ could be thus estimated to

$$\frac{8}{500{,}000} \times \frac{5}{160} = \frac{1}{2 \times 10^6}\text{ radians.}$$

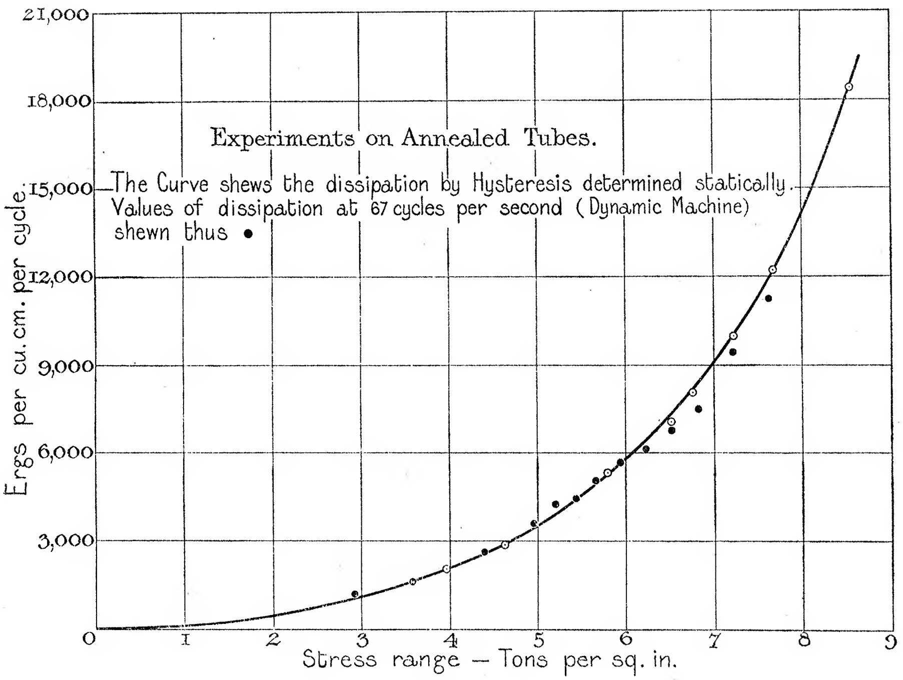


Fig. 9.

Assuming $C = 12 \times 10^6$ lbs. per square inch, the hysteresis stress difference could be calculated to within

$$12 \times 10^6 \times \frac{1}{2 \times 10^6} = 6 \text{ lb. per square inch.}$$

For a stress range of 7·67 tons per square inch the hysteresis stress difference is 0·109 ton per square inch (244 lb. per square inch) so that by means of the apparatus described above the width of the loop at the mid-point for this range can be estimated correct to within $2\frac{1}{2}$ per cent. of itself.

The above work was carried out at the Engineering Laboratory, Cambridge, and I wish to express my thanks to Prof. B. Hopkinson for his kind help and inspiring interest.